LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – NOVEMBER 2007

MT 5501 - REAL ANALYSIS

AB 17

Date : 26/10/2007 Dept. No. Max. : 100 Marks
Time : 9:00 - 12:00

SECTION – A

(10 x 2 = 20 marks)

Answer ALL questions.

1. Define Countable and uncountable sets.
2. State Cauchy – Schwarz inequality.
3. Prove that the intersection of an arbitrary collection of closed sets in a metric space *X* is closed in *X*.
4. Prove that a compact subset of a metric space is bounded.
5. Define a Cauchy sequence in a metric space.
6. Define the right hand and left hand limits of a function *F* at *C*.
7. If F is differentiable at *C*, then prove that *F* is continuous at *C*.
8. State Rolle's theorem.
9. State the linearity property of Riemann – Stieltjes integral.
10. Define limit superior of a real sequence.

SECTION – B

(5 x 8 = 40 marks)

Answer any FIVE questions.

11. State and prove Minkowski's inequality.
12. Prove that every subset of a countable set is countable.
13. If Y is a subspace of a metric space (*X*, *d*), then prove that a subset *A* of *Y* is open in *Y* if and only if $A = Y \cap G$ for some set *G* open in *X*.
14. Prove that a closed subset of a compact metric space is compact.
15. Prove that the Euclidean space R^K is complete.
16. Let (X,d_1) and (Y,d_2) be metric spaces, *X* be compact and $F : X \to Y$ be continuous on *X*. Show that *F* is uniformly continuous on *X*.
17. State and prove intermediate value theorem for derivatives.
18. Let $\{a_n\}$ be a real sequence. Then prove that
    - i) $\{a_n\}$ converges to ℓ if and only if $\liminf a_n = \limsup a_n = \ell$
    - ii) $\{a_n\}$ diverges to $+\infty$ if and only if $\liminf a_n = +\infty$

SECTION – C

(2 x 20 = 40 marks)

Answer any TWO questions.

19. a) Prove that the set *R* is uncountable. (8)

b) If *F* is a countable collection of pair wise disjoint countable sets, then prove that $\bigcup_{F \in F} F$ is countable. (7)

c) If *A* is a countable set and *B* an uncountable set, then prove that *B*-*A* is similar to *B*. (5)

20. a) Prove that every bounded and infinite subset of *R* has atleast one accumulation point. (10)

b) Let *S* and *T* be subsets of a metric space (*X*,*d*). Prove that S^1 is a closed set in *X*. (10)

21. a) Prove that in a metric space every convergent sequence is Cauchy. (6)

b) Let (X, d_1) and (Y, d_2) be metric spaces. Then prove that a map. $F : X \to Y$ is continuous on *X* if and only if F^{-1}(G) is open in X for every open set *G* in *Y*. (14)

22. a) State and prove Taylor's theorem. (10)

b) Let $F \in R(\alpha)$ on [a,b]. Then prove that $\alpha \in R(F)$ on [a,b] and

$$\int_a^b f d\alpha + \int_a^b \alpha df = F(b) - F(a)\alpha(b) - \int F(a)\alpha(a). \qquad (10)$$

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